Crossover behaviour in one-dimensional disordered systems in external electric fields

1989 J. Phys.: Condens. Matter 1 123

(http://iopscience.iop.org/0953-8984/1/1/011)

J. Phys.: Condens. Matter 1 (1989) 123–133. Printed in the UK

Crossover behaviour in one-dimensional disordered systems in external electric fields

J C Flores, H R Jauslin and C P Enz
Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland

Received 11 April 1988

Abstract. A method formulated by Felderhof is extended to the treatment of the conduction problem of disordered systems in external electric fields. We apply it to calculate averages and fluctuations for two models: model A consists of a sequence of δ-barriers with random position and amplitude, and model B of a sequence of square barriers with random position, height and width. The two models have qualitatively different behaviours, which can be explained by the fact that the square barriers are bounded. We calculate the ratios of incident to transmitted energy and of incident to transmitted current, which are expressed in terms of the scattering coefficients. The electric field produces a power-law dependence of the averages of those quantities, as opposed to the exponential dependence found for zero field. Further, in model A, there are two qualitatively different regimes at finite fields. Two different critical fields defined in terms of the behaviour of the transmission coefficient have been proposed in the literature. The study of the transmission of energy and current allows us to give them a well defined physical interpretation, as the fields at which the energy and current transmissions, respectively, switch from tending to zero to tending to infinity. In the zero-field case, the fluctuations are known to dominate exponentially over the averages. In the presence of an electric field, we find that in model A the fluctuations still dominate, but only algebraically, whereas in model B the relative fluctuations saturate to a constant.

1. Introduction

The problem of electrical conduction in one-dimensional disordered systems can be treated as a quantum mechanical scattering problem. From the transmission and reflection coefficients an expression for the resistance was deduced in [1]. For vanishing external electric field the resistance grows exponentially with increasing number of scatterers, reflecting the localisation of the states. It is known that this resistance has an anomalous statistical behaviour, in which the fluctuations dominate the averages exponentially. These results have been well established, both numerically and analytically [2–7].

The situation in the presence of a finite electric field is far less understood [8–15]. Several models [11–15] have been studied in the literature. In [10], it was proved that in random-amplitude Kronig–Penney models in a constant electric field there are two regimes in the spectrum: for small fields all the states are algebraically localised, while for large fields they are all delocalised. The method in [10] does not yield the value of the critical field. Previously, numerical studies of the scattering properties of that model had been made [9], and similar conclusions for the transmission coefficients had been reached. Two different criteria were proposed to define the critical field separating the two regimes in terms of the asymptotic behaviour of the transmission coefficients. However, the physical interpretation and the relation to the crossover in the spectral properties was unclear. In [14] an approximate analytical expression was found for the quotient of reflection and transmission coefficient and related to the numerical results in [9]. In [11] a class of smooth and bounded random potentials in a constant electric field was considered and it was proved that the spectrum is absolutely continuous. A model of a different type, in which the random potential is taken to be white noise, was treated in [15].

In this paper, we extend Felderhof's method developed in [7] to the treatment of models with an external electric field. It allows us to calculate averages and fluctuations for the scattering problem in models with barriers of different shapes. We treat, in particular, two models: model A, which is similar to the above-mentioned random-amplitude Kronig–Penney model, with the extension that also the positions of the δ-barriers are random; model B, in which the potential consists of square barriers with random position, height and width. We define two critical fields in terms of the transmission of energy and current across the barriers. They are equivalent to those proposed in [9] and provide a well defined physical interpretation.

The potentials which we consider consist of a sequence of N scattering barriers, superposed on a series of steps of height F representing the potential of the electric field. The Hamiltonians are of the form

$$H = -\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \sum_{j=1}^{N} V_j(x - x_j) - F \sum_{j=1}^{N} \theta(x - x_j - d_j) \tag{1.1}$$

where x_j is the centre of the jth barrier and $2d_j$ its width. We consider in particular two models: the δ-potential (model A) where

$$V_j(x - x_j) = v_j \delta(x - x_j) \qquad d_j \equiv 0 \tag{1.2}$$

and the square barrier (model B) where

$$V_j(x - x_j) = \begin{cases} v_j & \text{for } -d_j \leqslant (x - x_j) \leqslant d_j \\ 0 & \text{elsewhere.} \end{cases} \tag{1.3}$$

The $\{x_j, d_j, v_j\}$ are random variables with probability density

$$P(x_1, d_1, v_1; \ldots, x_N, d_N, v_N) = \delta(x_1) \prod_{j=2}^{N} f(x_j - x_{j-1}) \prod_{i=1}^{N} g(v_i) h(d_i) \tag{1.4}$$

where f, g and h are arbitrary probability densities which satisfy the following properties.

(i) Their first two moments are finite.

(ii) The supports of f and h are such that the barriers do not overlap.

(iii) For model B the v_j are bounded (less than k_0^2).

(iv) We choose the scales such that $\langle x_j - x_{j-1} \rangle = 1$, and we take f to be symmetric around 1.

With these conditions the model represents a sequence of random barriers with an external electric field of average strength F.

In § 2, we formulate the scattering problem and define the variables which describe the transmission properties of current and energy that we use later for the interpretation of the results. In § 3, we describe the basic features of Felderhof's method adapted to models with a finite external electric field. In § 4, we present the results concerning the averages for models A and B and, in § 5, we discuss the fluctuations.

2. Scattering coefficients

We treat the scattering problem of a particle of momentum k_0 incident from the left and scattered into a reflected and a transmitted part. The transmission coefficient T and reflection coefficients R are calculated using plane waves. On the left of the barriers, we have the incident and reflected waves

$$\psi_- = \exp(\mathrm{i}k_0 x) + r\exp(-\mathrm{i}k_0 x) \tag{2.1}$$

and to the right, after N barriers, the transmitted wave

$$\psi_+ = t\exp(\mathrm{i}k_N x) \tag{2.2}$$

where

$$k_N = \sqrt{k_0^2 + FN}. \tag{2.3}$$

The transmission coefficient T and reflection coefficient R are related to the amplitudes by

$$R = |r|^2 \qquad T = (k_N/k_0)|t|^2. \tag{2.4}$$

The conservation of current is expressed by

$$R + T = 1. \tag{2.5}$$

Between two barriers, ψ can be written as

$$\psi = A_j\exp(\mathrm{i}k_j x) + B_j\exp(-\mathrm{i}k_j x) \tag{2.6}$$

with k_j defined by (2.3). Successive pairs of coefficients are related by a transfer matrix:

$$\begin{pmatrix} A_{j+1} \\ B_{j+1} \end{pmatrix} = \mathbf{M}_j \begin{pmatrix} A_j \\ B_j \end{pmatrix} \tag{2.7}$$

which can be written as

$$\mathbf{M}_j = \mathbf{G}_2^*(x_j, k_{j+1})\mathbf{K}_2(j)\mathbf{G}_2(x_j, k_j) \tag{2.8}$$

where

$$\mathbf{G}_2(x, k) = \begin{pmatrix} \exp(\mathrm{i}kx) & 0 \\ 0 & \exp(-\mathrm{i}kx) \end{pmatrix}$$

$$\mathbf{K}_2(j) = \begin{pmatrix} \alpha_j & \beta_j \\ \beta_j^* & \alpha_j^* \end{pmatrix}. \tag{2.9}$$

The α_j and β_j can be calculated explicitly for our two models: for the δ-potential model A,

$$\begin{aligned}\alpha_j &= \tfrac{1}{2}[(1 + k_j/k_{j+1}) + \mathrm{i}(v_j/k_{j+1})]\\ \beta_j &= \tfrac{1}{2}[(1 - k_j/k_{j+1}) + \mathrm{i}(v_j/k_{j+1})]\end{aligned} \tag{2.10}$$

and, for the square-barrier model B,

$$\begin{aligned}\alpha_j = {} & \{\exp[-\mathrm{i}d_j(k_j + k_{j+1})]/4\hat{k}_j k_{j+1}\}[2\hat{k}_j(k_{j+1} + k_j)\\ & \times \cos(2d_j\hat{k}_j) + 2\mathrm{i}(\hat{k}_j^2 + k_j k_{j+1} \sin(2d_j\hat{k}_j)]\\ \beta_j = {} & \{\exp[\mathrm{i}d_j(k_j - k_{j+1})]/4\hat{k}_j k_{j+1}\}[2\hat{k}_j(k_{j+1} - k_j)\\ & \times \cos(2d_j\hat{k}_j) + 2\mathrm{i}(\hat{k}_j^2 - k_j k_{j+1} \sin(2d_j\hat{k}_j)]\end{aligned} \tag{2.11}$$

where

$$\hat{k}_j = \sqrt{k_j^2 - v_j}. \tag{2.12}$$

We should remark that the present method can be applied to symmetric barriers of any shape for which one can calculate α and β. It allows us to calculate the averages and fluctuations of the ratio R/T.

In the case of vanishing electric field, R/T is directly related to the resistance of the sample by Landauer's formula [1]. In the presence of a finite electric field, we can still interpret R/T in terms of the ratios between incoming and transmitted energy and momentum. To make this explict, we consider the scattering of a wavepacket with amplitude $f(k)$. For $t \to -\infty$, we have

$$\psi(x, t) \underset{t\to -\infty}{\to} \theta(x_{\mathrm{L}} - x) \int_0^\infty \mathrm{d}k\, f(k) \exp[\mathrm{i}(kx - Et)] \tag{2.13}$$

and, for $t \to +\infty$,

$$\begin{aligned}\psi(x, t) \underset{t\to +\infty}{\to} {} & \theta(x_{\mathrm{L}} - x) \int_0^\infty \mathrm{d}k\, f(k) r(k) \exp[-\mathrm{i}(kx + Et)]\\ & + \theta(x - x_{\mathrm{R}}) \int_0^\infty \mathrm{d}k\, f(k) t(k) \exp[\mathrm{i}(\sqrt{k^2 + FN}x - Et)]\end{aligned} \tag{2.14}$$

where x_{L} and x_{R} are some points to the left and to the right of the barriers, respectively, and $f(k)$ is sharply peaked at around k_0. We define the incident, transmitted and reflected kinetic energy averages as

$$\begin{aligned}E_{\mathrm{kin}}^{(\mathrm{I})} &:= \lim_{t\to -\infty} \langle\psi|p^2|\psi\rangle = k_0^2\\ E_{\mathrm{kin}}^{(\mathrm{T})} &:= \lim_{t\to +\infty} \langle\psi|\theta(x - x_{\mathrm{R}})p^2\theta(x - x_{\mathrm{R}})|\psi\rangle = |t|^2(k_N/k_0)k_N^2 = Tk_N^2\\ E_{\mathrm{kin}}^{(\mathrm{R})} &:= \lim_{t\to +\infty} \langle\psi|\theta(x_{\mathrm{L}} - x)p^2\theta(x_{\mathrm{L}} - x)|\psi\rangle = |r|^2 k_0^2 = Rk_0^2.\end{aligned} \tag{2.15}$$

The (approximate) equalities on the right are a direct consequence of the strongly peaked $f(k)$. For the currents ($\sim \langle\psi|p|\psi\rangle$), we obtain analogously

$$J^{(\mathrm{I})} = k_0 \qquad J^{(\mathrm{T})} = Tk_N \qquad J^{(\mathrm{R})} = Rk_0. \tag{2.16}$$

We shall consider the ratios

$$E^{(\mathrm{I})}_{\mathrm{kin}}/E^{(\mathrm{T})}_{\mathrm{kin}} = (R/T+1)(k_0^2/k_N^2) \tag{2.17}$$

and

$$J^{(\mathrm{I})}/J^{(\mathrm{T})} = (R/T+1)(k_0/k_N). \tag{2.18}$$

These relations will allow us to interpret the crossover phenomena encountered in model A.

3. Felderhof's method

We summarise here the main steps of Felderhof's method, stressing the modifications needed to treat models with an external electric field. For more details the reader is referred to [7]. The main result for the averages is the following expression:

$$\left\langle\frac{R}{T}\right\rangle \simeq \frac{1}{2}\frac{k_N}{k_0}\left(\prod_{j=1}^{N}\langle|\alpha_j|^2+|\beta_j|^2\rangle-\frac{k_0}{k_N}\right) \tag{3.1}$$

where α_j and β_j are defined in equations (2.8) and (2.9). The validity of this formula is not restricted to our two models. It applies to any barrier shape with a transfer matrix that admits the decomposition (2.8) and (2.9). This includes, for example, any symmetric barrier of finite width.

We should remark that, in the case $F=0$, $\langle|\alpha_j|^2\rangle$ and $\langle|\beta_j|^2\rangle$ are independent of j, and one obtains immediately the exponential behaviour $\langle R/T\rangle \sim \exp(\gamma N)$ for any shape of the barriers.

Equation (3.1) is obtained as follows. We consider a pair of auxiliary variables $a_1(x)$ and $a_2(x)$, which are defined piecewise in each interval $\{I_j\}_{j=0}^{N} := \{x \leqslant x_1, x_1 < x \leqslant x_2, \ldots, x_{N-1} < x \leqslant x_N, x_N < x\}$ (i.e. between centres of consecutive barriers) by

$$\left.\begin{aligned} a_1(x) &:= [\exp(-\mathrm{i}k_jx)/(1-|r|^2)[(1-r)A_j^* + (1-r^*)B_j] \\ a_2(x) &:= \mathrm{i}[\exp(-\mathrm{i}k_jx)/(1-|r|^2)][(1+r)A_j^* - (1+r^*)B_j] \end{aligned}\right\} \quad \text{for } x \in I_j \tag{3.2}$$

where k_j, A_j, B_j take the values corresponding to the interval I_j. Note that $a_1(x)$ and $a_2(x)$ are discontinuous at the points x_j, $j=1,\ldots,N$. Their values at those points are defined as the limit from the left.

We can now express the quantity $|r|^2/|t|^2$ in terms of $a_1(x)$ and $a_2(x)$ evaluated at an arbitrary point $x > x_N$. We proceed as follows. Since $A_N = t$ and $B_N = 0$, the evaluation of (3.2) at $x > x_N$ gives

$$\begin{aligned} a_1(x) &= [\exp(-\mathrm{i}k_Nx)/(1-|r|^2)](1-r)t^* \\ a_2(x) &= \mathrm{i}[\exp(-\mathrm{i}k_Nx)/(1-|r|^2)](1+r)t^*. \end{aligned} \tag{3.3}$$

Inverting (3.3) and using $|r|^2 + |t|^2k_N/k_0 = 1$, we obtain

$$|r|^2/|t|^2 = \tfrac{1}{2}(k_N/k_0)^2[\Gamma(x) - k_0/k_N] \qquad \text{for } x > x_N \tag{3.4}$$

with

$$\Gamma(x) := \tfrac{1}{2}[|a_1(x)|^2 + |a_2(x)|^2]. \tag{3.5}$$

Note that $\Gamma(x)$ does not depend on the particular x, provided that $x > x_N$.

For the evaluation of $\Gamma(x)$, we proceed as follows. We know (from $A_0 = 1$, $B_0 = r$ and (3.2)) that, for $x \leqslant x_1$,

$$a_1(x) = \exp(-\mathrm{i}k_0 x) \qquad a_2(x) = \mathrm{i}\exp(-\mathrm{i}k_0 x). \tag{3.6}$$

From their definition (3.2) and the known transformation of the A_j, B_j, k_j from one interval to the next ((2.7) and (2.8)), we deduce the transformation of a_1 and a_2. For $x_j < x \leqslant x_{j+1}$, we obtain the relation

$$\begin{pmatrix} a_\nu^*(x) \\ a_\nu(x) \end{pmatrix} = \mathbf{G}_2(x - x_j)\mathbf{K}_2(j)\begin{pmatrix} a_\nu^*(x_j) \\ a_\nu(x_j) \end{pmatrix} \qquad \nu = 1, 2 \tag{3.7}$$

i.e. $a_\nu(x)$ for x in the interval I_j is expressed in terms of a_ν at x_j, the right border of the interval I_{j-1}. Thus, by iteration, we get, for $x > x_N$,

$$\begin{pmatrix} a_\nu^*(x) \\ a_\nu(x) \end{pmatrix} = \mathbf{G}_2(x - x_N)\mathbf{K}_2(N)\mathbf{G}_2(x_N - x_{N-1}) \times \mathbf{K}_2(N-1)\ldots\mathbf{G}_2(x_2 - x_1)\mathbf{K}_2(1)\begin{pmatrix} a_\nu^*(x_1) \\ a_\nu(x_1) \end{pmatrix}. \tag{3.8}$$

Instead of calculating the $a_\nu(x)$ for $x > x_N$ by this iteration and then calculating Γ, it is more convenient to write an iteration directly for Γ. However, insertion of (3.7) into (3.5) shows that, as well as the desired term Γ, the iteration produces terms of the forms $a_\nu a_\mu$, $a_\nu^* a_\mu^*$ and $a_\nu^* a_\mu + a_\nu a_\mu^*$. One is thus led to consider the transformation of vectors with three components

$$\begin{pmatrix} a_\nu^*(x)a_\mu^*(x) \\ a_\nu^*(x)a_\mu(x) + a_\nu(x)a_\mu^*(x) \\ a_\nu(x)a_\mu(x) \end{pmatrix} = \mathbf{G}_3(x - x_j)\mathbf{K}_3(j)\begin{pmatrix} a_\nu^*(x_j)a_\mu^*(x_j) \\ a_\nu^*(x_j)a_\mu(x_j) + a_\nu(x_j)a_\mu^*(x_j) \\ a_\nu(x_j)a_\mu(x_j) \end{pmatrix} \tag{3.9}$$

where $\mathbf{G}_3$ and $\mathbf{K}_3$ are now 3×3 matrices:

$$\mathbf{G}_3(x - x_j) = \begin{pmatrix} \exp[2\mathrm{i}k_j(x - x_j)] & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \exp[-2\mathrm{i}k_j(x - x_j)] \end{pmatrix} \tag{3.10}$$

$$\mathbf{K}_3(j) = \begin{pmatrix} \alpha_j^{*2} & \alpha_j^*\beta_j^* & \beta_j^{*2} \\ 2\alpha_j^*\beta_j & |\alpha_j|^2 + |\beta_j|^2 & 2\alpha_j\beta_j^* \\ \beta_j^2 & \alpha_j\beta_j & \alpha_j^2 \end{pmatrix}. \tag{3.11}$$

The quantity $\Gamma(x)$ for $x > x_N$ can be expressed as

$$\Gamma(x) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}^{\mathrm{T}} \mathbf{K}_3(N)\mathbf{G}_3(x_N - x_{N-1})\mathbf{K}_3(N-1)\ldots\mathbf{G}_3(x_2 - x_1)\mathbf{K}_3(1)\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}. \tag{3.12}$$

Since the probabilities of each barrier are independent of the others, the average factorises:

$$\langle \Gamma(x)\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}^{\mathrm{T}} \langle \mathbf{K}_3(N)\rangle\langle \mathbf{G}_3(x_N - x_{N-1})\rangle\langle \mathbf{K}_3(N-1)\rangle \ldots \langle \mathbf{G}_3(x_2 - x_1)\rangle\langle \mathbf{K}_3(1)\rangle \begin{pmatrix}0\\1\\0\end{pmatrix}. \tag{3.13}$$

Up to this point the calculation was exact. Now we make the following approximation: for

$$k_j(x_j - x_{j+1}) \gg 1 \tag{3.14}$$

we can put

$$\langle \mathbf{G}_3(x_j - x_{j+1})\rangle \simeq \begin{pmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}. \tag{3.15}$$

Note that, since $k_j^2 = k_0^2 + F_j$, the condition (3.14) will certainly be satisfied in the asymptotic regime of large j. Equation (3.13) then gives

$$\langle \Gamma(x)\rangle \simeq \prod_{j=1}^{N} \langle (K_3(j))_{22}\rangle \equiv \prod_{j=1}^{N} \langle |\alpha_j|^2 + |\beta_j|^2\rangle. \tag{3.16}$$

Finally, putting together equations (2.4), (3.4) and (3.16), we obtain the result (3.1).

4. Asymptotic behaviour of $\langle R/T\rangle$

We can now evaluate equation (3.1) for models A and B and discuss the asymptotic dependence for a large number N of barriers.

4.1. Model A

For model A, equation (3.16) gives

$$\langle \Gamma\rangle \simeq \prod_{j=1}^{N} \left(1 + \frac{\sigma^2 - F}{2[k_0^2 + (j+1)F]}\right) \tag{4.1}$$

where $\sigma^2 := \langle v_j^2\rangle$ (independent of j). The asymptotic dependence on N can be evaluated using the following formula:

$$\prod_{j=1}^{N} \left(1 + \frac{\omega}{j + c}\right) \underset{N\to+\infty}{\longrightarrow} g(\omega, c) N^{\omega} \tag{4.2}$$

where g is an N-independent function. Applied on (4.1), it gives

$$\langle \Gamma\rangle \underset{N\to+\infty}{\longrightarrow} g(k_0, F, \sigma^2) N^{(\sigma^2 - F)/2F} \tag{4.3}$$

(for $F \neq 0$). Insertion into (3.1) gives

$$\langle R/T\rangle \underset{N\to+\infty}{\longrightarrow} \tfrac{1}{2}(k_N/k_0) g N^{(\sigma^2 - F)/2F} \sim N^{\sigma^2/2F}. \tag{4.4}$$

Since the approximation (3.15) becomes exact as $N \to \infty$, we expect (4.4) to be the exact asymptotic behaviour. The effect of the electric field is thus a first crossover to power-law dependence of $\langle R/T \rangle$ from the exponential behaviour found for $F = 0$. This is in agreement with the results in [9], [10] and [14] for the random-amplitude Kronig–Penney model.

Equation (4.4) suggests the existence of a second crossover between two qualitatively different behaviours, depending on the value of σ^2/F. Several definitions for the transition point have been proposed [9]. In [10], it was proved that a crossover exists in the nature of the spectrum at some critical field F_c. For $F > F_c$ the spectrum is absolutely continuous, while for $F < F_c$ it is a pure point spectrum (weak localisation; the corresponding eigenfunctions decrease algebraically as opposed to exponentially as found for $F = 0$). This treatment does not yield the value of the critical field F_c. (More precisely, it was proved only that for F smaller than some F_a the spectrum is a pure point spectrum, while for F larger than some F_b it is absolutely continuous. The conjecture is that $F_a = F_b \equiv F_c$.) There are two differences between the model in [10] and the present one.

(i) In [10], δ-barriers with fixed positions x_j, random amplitudes v_j and a constant electric field were considered, whereas here both x_j and v_j are random and the field is represented by a staircase. We do not expect this difference to affect the results (note, for example, that the particular distribution of the positions x_j does not enter in equation (4.4)).

(ii) In [10] the spectral problem for a set of barriers and a field that extended from $-\infty$ to $+\infty$ was treated, while in our case we have free incoming, transmitted and reflected waves.

The two models are close enough to suggest that the spectral properties in [10] must be somehow related to the scattering properties of our model A. In particular, one could expect that the crossover in the spectrum manifests itself in the N-dependence of the transmission and reflection coefficients.

In [9], two possible definitions of a critical field were proposed in terms of the scattering coefficients.

(i) F_{c1} was defined as the field at which $T \sim N^{-1}$, which in the present treatment is $F_{c1} = \sigma^2/2$.

(ii) F_{c2} was defined as the field at which $|t|^2 \sim N^{-1}$, which gives $F_{c2} = \sigma^2$.

However, it was unclear what physical phenomenon they should reflect, and the question of which of the two was the 'correct' crossover field remained unanswered. The idea, for example, that the asymptotic behaviour of t is equal to that of the eigenfunctions is not *a priori* justified, since the two quantities are defined with completely different boundary conditions. Below we shall give a direct physical meaning to the two proposed values in terms of transmitted energy and current. We conclude that, depending on which quantity one measures, the crossover will appear at one or the other critical field.

We first consider the ratio of incoming to transmitted kinetic energy of a wavepacket as defined in § 1. Insertion of (4.4) into (2.17) yields

$$\langle E_{\mathrm{kin}}^{(\mathrm{I})}/E_{\mathrm{kin}}^{(\mathrm{T})} \rangle \sim N^{(\sigma^2 - 2F)/2F}. \tag{4.5}$$

This relation defines a crossover in the energy transmission at a critical field

$$F_{c1} = \tfrac{1}{2}\sigma^2. \tag{4.6}$$

For the corresponding ratio of the currents, we obtain (from (2.18) and (4.4))

$$\langle J^{(\mathrm{I})}/J^{(\mathrm{T})}\rangle \sim N^{(\sigma^2-F)/2F} \tag{4.7}$$

which determines the critical field for the current as

$$F_{c2} = \sigma^2. \tag{4.8}$$

Thus the two critical fields proposed in [9] have a definite physical interpretation. Depending on which quantity one considers, the crossover will appear at one or the other critical field.

4.2. Model B

The corresponding calculations for model B give

$$\langle \Gamma \rangle \sim N^{-1/2} \tag{4.9}$$

and

$$\langle R/T\rangle = (k_0/k_N)\langle |r|^2/|t|^2\rangle \sim (k_0/k_N)N^{1/2} \sim \text{constant} \tag{4.10}$$

and for the energy and current transmission ratios

$$\langle E^{(\mathrm{I})}_{\mathrm{kin}}/E^{(\mathrm{T})}_{\mathrm{kin}}\rangle \sim N^{-1} \tag{4.11}$$

$$\langle J^{(\mathrm{I})}/J^{(\mathrm{T})}\rangle \sim N^{-1/2}. \tag{4.12}$$

This is the same behaviour which one would obtain in absence of barriers. The asymptotic behaviour is completely dominated by the electric field. This result shows that the crossover is very specific of model A. The relevant difference is that the potential is bounded in model B and unbounded in model A. The incoming particles 'feel' the δ-potentials irrespective of how high their energy is. One knows also [11] that the spectrum for any bounded potential superposed on a constant electric field is always absolutely continuous. From these arguments, we expect equations (4.9)–(4.12) to apply also for models with bounded barriers of any shape.

5. Fluctuations

It is well known [2, 3, 6] that, in the case without the electric field, $|r|^2/|t|^2$ has an anomalous statistical behaviour, in that the fluctuations increase with increasing N exponentially faster than the averages.

In the presence of a finite electric field, we find qualitatively different fluctuation properties between models A and B. The relative fluctuation Δ is defined as

$$\Delta^2 \simeq (\langle R^2/T^2\rangle - \langle R/T\rangle^2)/\langle R/T\rangle^2 = (\langle \Gamma^2\rangle - \langle \Gamma\rangle^2)/(\langle \Gamma\rangle - k_0/k_N)^2. \tag{5.1}$$

The quantity $\langle \Gamma^2\rangle$ can be evaluated [7] by a similar method to the method that we used for $\langle \Gamma\rangle$. One obtains the expression

$$\langle \Gamma^2\rangle := \tfrac{2}{3}\prod_{j=1}^{N}\langle |\alpha_j|^4 + |\beta_j|^4 + 4|\alpha_j|^2|\beta_j|^2\rangle + \tfrac{1}{3}\left(\frac{k_0}{k_N}\right)^2. \tag{5.2}$$

For model A, we obtain

$$\langle \Gamma^2\rangle \sim N^{3\sigma^2/2F} - 1 \tag{5.3}$$

and therefore

$$\Delta^2 \sim N^{\sigma^2/2F} \to \infty. \tag{5.4}$$

Thus, in the presence of F, the fluctuations still dominate over the averages but only algebraically (as opposed to exponentially in the zero-field case).

For model B, we obtain

$$\langle \Gamma^2 \rangle \sim N^{-1} \tag{5.5}$$

which combined with (4.9) and (5.1) gives

$$\Delta^2 \sim \text{constant}. \tag{5.6}$$

The fluctuations in model B do not dominate over the averages.

6. Conclusion

We have obtained analytic expressions for the asymptotics of the average and fluctuations of the quantity R/T for two models representing disordered one-dimensional systems in an external electric field F. Model A consists of (unbounded) δ-barriers, and model B of bounded square barriers. This difference is crucial and it produces qualitatively different results. For $F \neq 0$, in model A, $\langle R/T \rangle$ has an algebraic asymptotic dependence on the number N of barriers, whereas in model B, it tends to a finite constant, in contrast with the exponential dependence found in both models in the zero-field case. Figure 1 summarises the behaviour of the exponents for the two models.

In the zero-field case, R/T is directly related to the resistivity by Landauer's formula. Its interpretation in the case with a finite electric field was less evident. We give it a direct physical interpretation by relating it to the transmission of energy and current. This allows us to interpret the two possible critical fields which were proposed in [9] for the crossover phenomena that are found in model A. The field F_{c1} was defined as the field at which $T \sim 1/N$ is equal to the field at which the transmission of energy for large N switches from tending to zero to tending to infinity; for model A, it is given by $F_{c1} = \sigma^2/2$. The field F_{c2} was defined by $|t|^2 \sim 1/N$ is equal to the field at which the transmission of current switches from tending to zero to tending to infinity; for model A, it is given

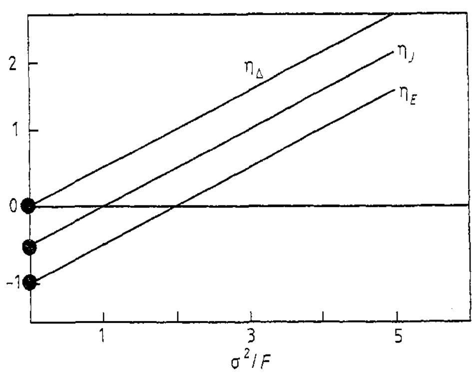

Figure 1. Exponents η_E, η_J and η_Δ defined respectively by the asymptotic behaviour of $\langle E^{(I)}_{\text{kin}}/E^{(T)}_{\text{kin}} \rangle \sim N^{\eta_E}$, $\langle J^{(I)}/J^{(T)} \rangle \sim N^{\eta_J}$ and $\Delta^2 \sim N^{\eta_\Delta}$: ——, exponents of model A as a function of σ^2/F; ●, exponents of model B.

by $F_{c2} = \sigma^2$. For model B, no such crossover is observed; for any finite field F the transmission of energy and current always increases with increasing N to infinity. This is a consequence of the boundedness of the barriers and may be related to the fact [11] that in this case all states are delocalised. This shows that the interesting crossover phenomena encountered with δ-potentials is by no means a general phenomenon of disordered systems subjected to an external electric field but is linked to the special properties of δ-potentials.

The relative fluctuations of R/T are known to diverge exponentially when $F = 0$. For $F \neq 0$ they still diverge in model A, but only algebraically. For model B, they do not diverge but tend to a constant.

Acknowledgment

This work has been partially supported by the Fonds National Suisse de la Recherche Scientifique.

References

[1] Landauer R 1970 *Phil. Mag.* **21** 863
[2] Anderson P W, Thouless D J, Abrahams E and Fisher D S 1980 *Phys. Rev.* B **22** 3519
[3] Abrahams E and Stephen M J 1980 *J. Phys. C: Solid State Phys.* **13** L377
[4] Andereck B S and Abrahams E 1980 *J. Phys. C: Solid State Phys.* **13** L383
[5] Erdös P and Herndon R C 1982 *Adv. Phys.* **31** 65
[6] Kumar N 1985 *Phys. Rev.* B **31** 5513
[7] Felderhof B U 1986 *J. Stat. Phys.* **43** 267
[8] Prigodin V N 1980 *Zh. Eksp. Teor. Phys.* **79** 2338 (Engl. Transl. 1984 *Sov. Phys.-JETP* **52** 1187)
[9] Soukoulis C M, José J V, Economou E N and Sheng Ping 1983 *Phys. Rev. Lett.* **50** 764
[10] Deylon F, Simon B and Souillard B 1984 *Phys. Rev. Lett.* **52** 2187; 1985 *Ann. Inst. Henri Poincaré* **42** 283
[11] Bentosela F, Carmona R, Duclos P, Simon B, Souillard B and Weber R 1983 *Commun. Math. Phys.* **88** 387
[12] Flores J, José J V and Monsivais G 1983 *J. Phys. C: Solid State Phys.* **16** L103
[13] Flores J, Mello P A and Monsivais G 1987 *Phys. Rev.* B **35** 2144
[14] Castello D, Caro A and Lopez A 1987 *Phys. Rev.* B **36** 3002
[15] Vijayagovindan G V, Jayannavar A M and Kumar N 1987 *Phys. Rev.* B **35** 2029